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Supply Chain Vertical and Horizontal Cooperation for Carbon Emission Reduction Considering Bullwhip Effect under the Carbon Tax Scheme

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Abstract

Considering the bullwhip effect, in a two-echelon supply chain consisting of one single supplier and multiple retailers, the vertical and horizontal cooperation game for carbon emission reduction is analyzed under carbon tax scheme. This paper investigates four different decision models: decentralized decision, vertical cooperation, horizontal cooperation and vertical and horizontal cooperation. After analyzing and comparing the optimal solutions of different models, it is found that the vertical and horizontal cooperation is always a dominant strategy for the supply chain in terms of both of cost and carbon emissions. Then, the collaborative condition of the vertical and horizontal cooperation game is analyzed. The result shows that the higher the fixed construction cost of warehouse is, the stronger the cooperation motivation of supply chain members is. However, the higher carbon tax rate will hinder the cooperation and bring greater carbon emissions. Furthermore, a cost allocation scheme is proposed based on proportion rule to achieve Pareto improvement for supplier and retailers. Although the vertical and horizontal cooperation cost game is not necessarily concave, it is permutationally concave game.

Keywords: carbon emission reduction, vertical and horizontal cooperation, bullwhip effect, cost allocation scheme

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INTRODUCTION

With low carbon economy becoming a new trend worldwide, reducing carbon emissions in supply chain has become a hot topic worldwide. The 2500 largest global enterprises account for more than 20% of carbon emissions, and most of the carbon emissions are generated by corporate operations associated with their supply chain (Jira and Toffel 2013). Due to many benefits, such as increased profits, reduced costs and decreased greenhouse gas emissions, cooperation has become a significant factor to improve the supply chain performance (Elena et al. 2015, Judith et al. 2013, Wei et al. 2015). The primary purpose of supply chain members' cooperation is improving profits and reducing costs, and the secondary aim is decreasing carbon emissions (Dai et al. 2017, Wahab et al. 2011). The selection sequence conforms to the assumption that the enterprise is rational. Therefore, the objective of this paper to link the cooperation strategy with the carbon emissions factor when the supply chain members employ cooperation strategy to decrease overall supply chain costs.

After Kyoto Protocol in 1997, many countries such as Finland, Sweden and Ireland has implemented carbon tax policy to reduce carbon emissions. Carbon tax mechanism has been proved to be an effective way to achieve emission reduction goals (Dong et al. 2017, Kuo et al. 2016, Presley et al. 2017, Rivers et al. 2015). Carbon policy implies that a firm has to pay for its carbon emissions into environment. The carbon emission cost will impact the decision-making of enterprises. Therefore, supply chain members employing cooperation strategy must consider carbon emissions cost.

In order to control warehousing more flexibly and effectively, lots of enterprises invest heavily in warehouses construction. For example, Meter Sbonwe Fashion & Accessories Limited, a giant enterprise of manufacturing and selling clothing in mainland China, build its own warehouses aiming to solve complex logistics distribution problem. Although having its own warehouse shows the firm's strong strength, warehouses construction expends great cost, especially building a stereoscopic warehouse. How to enjoy the advantages of self building warehouse and avoid high

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cost expenditure becomes a key issue which companies have to face. Multiple enterprises using a warehouse together and managing inventory jointly, which is also known as jointly managed inventory (JMI), is an effective way to solve the dilemma.

If supplier only considers the order quantity of contiguous downstream of supply chain when makes ordering/manufacturing decision, the supplier is incline to order/manufacture more products than actual demand of market aiming to meet demand fluctuation, which is known as bullwhip effect. Bullwhip effect is very common in the supply chain operation and harmful to the performance of supply chain, such as increasing channel inventory, adding cost and decreasing customer service level (Devika et al. 2016, Disney et al. 2006). Bullwhip effect will occur when the upstream and downstream enterprises of supply chain manage their inventory respectively, and does not share demand information of market. Supplier and retailers manage inventories jointly and share demand information of market in JMI mode. Hence, JMI is a useful method to alleviate bullwhip effect.

JMI is an inventory management mode which is developed on the basis of vendor managed inventory (VMI). It overcomes the drawbacks of VMI, such as unequal position of supplier and retailer, crisis of trust and difficulty in distributing profit. Therefore, it is of great theoretical and practical significance to study the problem of supply chain members' cooperation for reducing carbon emissions in JMI mode. Motivated by the above facts, this paper considers a supply chain which consists of one single supplier and multiple retailers under carbon tax scheme. The supply chain members employ JMI strategy to decrease cost and carbon emissions considering construction cost of warehouse. We propose three modes of cooperation among supply chain members. The first is supplier and retailer jointly managing inventories, namely vertical cooperation. The second is multiple retailers jointly managing inventories, namely horizontal cooperation. The third is supplier and retailers jointly managing inventories, namely vertical and horizontal cooperation. This paper addresses the following research questions using four models of decision-making.

(a) Which cooperation strategy works better for decrease cost and carbon emissions? The vertical and horizontal cooperation is a dominant strategy which not only decrease overall supply chain's cost, but also reduce carbon emissions in supply chain.

(b) Is there a reasonable cost allocation scheme to achieve Pareto improvement when supplier and retailers manage inventories jointly? We present a cost allocation scheme based on cost sharing rule proposed by Meca et al. (2004) and prove its effectiveness.

In addition, there is an interesting finding, that is, higher carbon tax will undermine cooperation, leading to an increase in carbon emissions in vertical and horizontal cooperation mode. However, the government implementing carbon tax aims to reduce carbon emissions primarily.

The reminder of the paper is organized as follows. Section 2 briefly reviews the related literature. In section 3, we investigate four decision models: supplier and retailers make decision respectively, namely decentralized decision (DD) model, vertical cooperation (VC) model, horizontal cooperation (HC) model and vertical and horizontal cooperation (VHC) model. In section 4, we discuss the condition for the cooperation and present a cost allocation scheme to achieve Pareto improvement in VHC mode. Section 5 provides numerical evidence which confirms the above Proposition. Concluding remarks and some directions for future research are presented in section 6. All proofs are presented in **Appendix**.

LITERATURE REVIEW

It has been widely recognized that supply chain members' cooperation is beneficial to reduce carbon emissions. There are two streams of literature related to our paper: Economic Order Quantity (EOQ) model and supply chain cooperation.

The EOQ model is an ordering mode which was first presented by Harris (1913) and is widely applied in practice. With increasing emphasis on carbon emission reduction, more and more scholars have integrated carbon emission into the extended study of the EOQ model. Hua et al. (2011) extended the EOQ model by integrating the impact of a carbon cost associated with both transport and storage under cap-and-trade scheme. They concluded that the optimal order quantity is between the traditional economic quantity and the quantity that minimizes carbon emissions. Chen et al. (2013) pointed out that changing the order quantity could reduce carbon emissions, and the decreasing amplitude in carbon emissions is larger than the increasing amplitude in cost. Dincer and Brian (2014) considered the integration problem of inventory management and transport planning under constraint of carbon emission. They compared four kinds of carbon policies and investigated their influence on optimal order quantity. He et al. (2015) addressed the issue of manufacture's optimal production lot size under carbon tax and cap-and-trade mechanism. The authors concluded that the two policies would lead to the same carbon emissions when the start up cost of production and unit holding cost are identical. Vincent and Laurent (2015) proposed a novel model that took into account the link between an inventory policy (EOQ), total carbon emissions, and both price and carbon emissions dependent demands. They concluded that carbon tax is conducive to reduce carbon emissions. Other related literatures on this topic include Bonney and Jaber (2011), Bouchery et al. (2012) and Battini et al. (2014). The above literatures investigated the enterprise optimal order quantity/ production lot size under carbon constraints. However, these studies were based on single enterprise, not taking overall supply chain into account. In fact, if a company changes its order quantity/production lot size, both the upstream and downstream of supply chain would be affected. Hence, it is more practical to study the issue of reducing carbon emission from the perspective of overall supply chain. At the same time, all the above literatures neglected the construction cost of warehouse, which is a very important cost for enterprises owning a warehouse.

The vertical cooperation among supply chain members to reduce carbon emissions has been studied and applied widely. Barari et al. (2012) developed an evolutionary game model to study the problem of vertical cooperation strategy in supply chain. They found that vertical cooperation strategy could not only improve revenue, but also reduce the carbon emissions in supply chain. Pioneering studies focused on pricing strategies, production decision, mechanisms of coordination and investment strategies. Zhang and Liu (2013) considered the pricing decisions in a three-level green supply chain system in which market demand correlates with product green degree. Ghosh and Shah (2012) investigated an apparel serial supply chain whose players initiate product "green." The authors proposed a two-part tariff contract to coordinate the green channel. Wang et al. (2016) modeled a game between a manufacturer and a retailer under low carbon environment. The authors concluded that the cost-sharing contract could promote cooperation between the manufacturer and the retailer. Cao et al. (2015) addressed the issue of cooperation in a two-echelon supply chain. They found that the cooperation could reduce carbon emissions and designed a revenue sharing contract to achieve Pareto improvement. Gao and Zhou (2015) analyzed the optimal investment strategies in decentralized and centralized decisions, considering a manufacturer and a retailer invest in improving existing production processes to reduce carbon emissions, and designed the supply chain cooperation mechanism by using Nash bargaining game. Du et al. (2015) focused on the impact of consumers' low carbon preferences on the supply chain, and adopted a new carbon sensitive demand function to analyze game model. They designed different supply chain coordination contracts. Ghosh and Shah (2015) discussed the vertical cooperation problem in green supply chain. They studied the impact of cost sharing contract on green decision. Other related literatures on this topic include Yang et al. (2014), Bazan et al. (2015), Lou et al. (2015), He et al. (2015) and Cheng et al. (2017). There are many literatures to study the vertical cooperation among supply chain members to reduce carbon emissions. However, the literatures that studied the horizontal cooperation are rare. Akshit et al. (2015) employed cloud computing technology to promote the cooperation between wholesalers in the beef supply chain. Elena et al. (2015) discussed horizontal cooperation in road transportation to reduce greenhouse gas emissions. There is also a small amount of literature to study vertical and horizontal cooperation among supply chain members. Yang et al. (2017) considered two competitive supply chains under the cap-and-trade scheme, each of which consists of one manufacturer and one retailer. They addressed the pricing and carbon emission reduction decisions considering vertical and horizontal cooperation. Li et al. (2017) examined the impacts of vertical and horizontal cooperation on the optimal decisions and performance of a low-carbon closed-loop supply chain. They concluded that the completely centralized model is best. The above literatures investigated vertical cooperation, horizontal cooperation and vertical and horizontal cooperation for carbon emission reduction respectively, but did not compare the three cooperation strategies in term of carbon emission reduction. At the same time, most of the extant literature did not consider the cooperation issue from the perspective of inventory management to reduce carbon emissions.

Our key contribution lies in the following third aspects. Firstly, we consider a supply chain consisting of one supplier and multiple retailers and develop four models to describe the different cooperation modes among supply chain members based on JMI strategy considering the construction cost of warehouse, which is ignored by the aforementioned literature. Secondly,

Table 1. Model notations

Parameters	
D_i	Annual demand of retailer i
μ	Carbon tax rate
C_{wa}	Annual construction cost of warehouse
F	Fixed construction cost of warehouse
B	Variable construction cost per product unit of warehouse
Q_m	Maximum inventory quantity in warehouse
K_s/K_r	Ordering cost per order of supplier/retailers
K_s^e/K_r^e	Carbon emissions per order linked to supplier's / retailers' order
H_s/H_i	Annual inventory holding cost per product unit of supplier/retailer i
H_s^e/H_i^e	Annual carbon emissions per product unit linked to supplier's/ retailer i's holding inventory
Decision variables	
Q_i	Ordering quantity of retailer i in DD/VC model
Q_t	Joint ordering quantity of retailers in HC/VHC model
W_i	Multiply of ordering quantity of retailer i in DD model
W_t	Multiply of joint ordering quantity of retailers in HC model
Other notations	
$N = \{1,2,\dots,n\}$	Set of retailers
s	Supplier
$i,j = 1,2,\dots,n$	Index to denote each retailer
E_i	Annual carbon emissions of retailer i in DD model
E_{si}	Annual total carbon emissions of retailer i and supplier in VC model
E_{rt}	Annual carbon emissions of retailers in HC model
E_s	Annual total carbon emissions of supplier
E_{sc}	Annual total carbon emissions of supply chain
C_i	Annual total cost of retailer i in DD model
C_{si}	Annual total cost of retailer i and supplier in VC model
C_{rt}	Annual total cost of retailers in HC model
C_s	Annual total cost of supplier
C_{sc}	Annual total cost of supply chain

we compare the effects of cost and carbon emission reduction of the vertical cooperation, horizontal cooperation and vertical and horizontal cooperation, which helps to provide valuable insights for supplier and retailers to choose the optimal cooperation strategy. The insights are different from those in the above mentioned literature. Finally, we propose a new cost allocation scheme to achieve Pareto improvement in VHC mode, which can promote the cooperation between the supplier and retailers effectively.

MODELS AND ANALYTICAL RESULTS

We consider a two-echelon supply chain which consists of one supplier and multiple retailers under carbon tax scheme. We utilize "s" and "N" to denote the supplier and retailers respectively. The supplier and retailers build their own warehouse and manage their own inventory respectively, when they don't employ JMI strategy, which can be labeled as decentralized decision (DD) mode. In order to reduce costs, the supplier and retailers intend to employ JMI strategy. With JMI strategy, the supplier and retailers just set up a warehouse where the annual total cost per product unit of holding inventory, which include holding cost and carbon emission cost generated by holding inventory, is the lowest, and manage inventories jointly. They can choose a cooperation mode among three alternatives. We label these modes separately as the vertical cooperation (VC) mode, horizontal cooperation (HC) mode and vertical and horizontal cooperation (VHC) mode. With the VC mode, each retailer and supplier manage inventories jointly, but retailers make decision respectively. With the HC mode, multiple retailers manage inventories jointly. There are not cooperation between supplier and retailers. With the VHC mode, supplier and retailers manage inventories jointly. For a better understanding of these models, we first summarize the basic notations of this paper as shown in **Table 1**.

Before set up the mathematical models, then, we need make some assumptions.

Hypothesis1. Ordering cost and carbon emission per order linked to order of all retailers are identical because they order the same product from the same supplier (Meca et al. 2004).

Hypothesis 2. The enterprise has lower annual carbon emissions generated by holding inventory, which has lower annual inventory holding cost and makes carbon emission control better, that is, for all $i, j \in (N, s)$, if $H_i \le H_j$, then $H_i^e \le H_j^e$.

If the enterprise does not have the lowest annual carbon emissions generated by holding inventory, which has the lowest annual inventory holding cost, then the annual total cost per product unit of holding inventory, which include holding cost and carbon emission cost generated by holding inventory, is not the least. However, the inventory is held in the warehouse in which the annual total cost per product unit of holding inventory is the lowest. Therefore, this assumption is for the simplicity of solving and analyzing the mathematical models.

Hypothesis 3. Depreciation fixed number of year of warehouse is 20 according to "Regulations for the Implementation of the PRC Enterprise Income Tax Law". So, we can use the function $C_{wa}(Q_m) = 0.05(F + BQ_m)$ to describe the annual construction cost of warehouse.

Hypothesis 4. The retailers and supplier share the market demand information in VC/VHC model, but not in DD/HC model.

Decentralized Decision Model

The supplier and retailers build their own warehouse and manage their own inventory respectively with DD mode. We assume that the supplier's ordering quantity is W_i times that of the retailer i, in order to meet demand fluctuation of retailer i. There are three types of cost involved. First, there is the annual construction cost of warehouse. Second, there is the annual ordering cost which includes carbon emission cost linked to order. Third, there is the annual inventory holding cost which includes carbon emission cost linked to holding inventory. The annual total carbon emissions and costs of the retailer i and supplier are as follows.

$$E_i(Q_i) = K_r^e\, D_i/Q_i + H_i^e\, Q_i/2 \tag{1}$$

$$E_s(W_i, Q_i) = \sum_{i\in N} K_s^e\, D_i/W_iQ_i + \sum_{i\in N} H_s^e\, W_iQ_i/2 \tag{2}$$

$$C_i(Q_i) = 0.05(F + BQ_i) + (K_r + \mu K_r^e)\, D_i/Q_i + (H_i + \mu H_i^e)\, Q_i/2 \tag{3}$$

$$C_s(W_i, Q_i) = 0.05(F + B\sum_{i\in N} W_iQ_i) + \sum_{i\in N}(K_s + \mu K_s^e)\, D_i/W_iQ_i + \sum_{i\in N}(H_s + \mu H_s^e)\, W_iQ_i/2 \tag{4}$$

In DD model, the retailers simultaneously and uncooperatively determine their own ordering quantity, and accordingly, the supplier determines the ordering multiply given ordering quantity of retailer i.

Proposition 1. In the DD model, the optimal solutions are as follows:

$$Q_i^{DD*} = \sqrt{2(K_r + \mu K_r^e)D_i/(0.1B + H_i + \mu H_i^e)} \tag{5}$$

$$W_i^{DD*} = \frac{1}{Q_i^{DD*}}\sqrt{2(K_s + \mu K_s^e)D_i/(0.1B + H_s + \mu H_s^e)} \tag{6}$$

For proof, see **Appendix A**.

The superscript $DD*$ denotes the optimal solutions under the DD mode. In the rest of this paper, we use a similar marketing method.

Substituting the optimal solutions into Equations (1-4), we derive all outcomes of this mode, as follows:

$$E_i^{DD*} = [K_r^e\sqrt{(0.1B + H_i + \mu H_i^e)/2(K_r + \mu K_r^e)} + H_i^e\sqrt{(K_r + \mu K_r^e)/2(0.1B + H_i + \mu H_i^e)}]\sqrt{D_i} \tag{7}$$

$$E_s^{DD*} = [K_s^e\sqrt{(0.1B + H_s + \mu H_s^e)/2(K_s + \mu K_s^e)} + H_s^e\sqrt{(K_s + \mu K_s^e)/2(0.1B + H_s + \mu H_s^e)}]\sum_{i\in N}\sqrt{D_i} \tag{8}$$

$$C_i^{DD*} = 0.05F + \sqrt{2(K_r + \mu K_r^e)(0.1B + H_i + \mu H_i^e)D_i} \tag{9}$$

$$C_s^{DD*} = 0.05F + \sqrt{2(K_s + \mu K_s^e)(0.1B + H_s + \mu H_s^e)}\sum_{i\in N}\sqrt{D_i} \tag{10}$$

$$E_{sc}^{DD*} = \sum_{i\in N}[K_r^e\sqrt{(0.1B + H_i + \mu H_i^e)/2(K_r + \mu K_r^e)} + H_i^e\sqrt{(K_r + \mu K_r^e)/2(0.1B + H_i + \mu H_i^e)}]\sqrt{D_i} + [K_s^e\sqrt{(0.1B + H_s + \mu H_s^e)/2(K_s + \mu K_s^e)} + H_s^e\sqrt{(K_s + \mu K_s^e)/2(0.1B + H_s + \mu H_s^e)}]\sum_{i\in N}\sqrt{D_i} \tag{11}$$

$$C_{sc}^{DD*} = 0.05(n+1)F + \sum_{i\in N}\sqrt{2(K_r + \mu K_r^e)(0.1B + H_i + \mu H_i^e)D_i} + \sqrt{2(K_s + \mu K_s^e)(0.1B + H_s + \mu H_s^e)}\sum_{i\in N}\sqrt{D_i} \tag{12}$$

Vertical Cooperation Model

Each retailer and supplier manages inventories jointly, but retailers make decision respectively with VC mode. Because of retailers and supplier sharing market demand information, the VC mode decrease the bullwhip effect effectively, with which the supplier does not need to place more orders to meet retailers' demand fluctuation. For the simplicity of solving and analyzing the mathematical model, we put retailers in order according to annual inventory holding cost per product unit, that is, retailer1's annual inventory holding cost per product unit is the minimal, and that of retailer n is the maximal. We assume that there are k retailers whose annual inventory holding cost per product unit is smaller than that of supplier, namely, $H_1 \le H_2 \le \cdots \ldots \le H_k \prec H_s \le H_{k+1} \le \cdots \ldots \le H_n$. Hence, there are two cases: (1) if $i \le k$, then the annual total carbon emissions and costs of the retailer i and supplier are as follows:

$$E_{si}(Q_i) = K_s^e\, D_i/Q_i + K_r^e\, D_i/Q_i + H_i^e\, Q_i/2 \quad (13)$$

$$C_{si}(Q_i) = 0.05(F + BQ_i) + (K_s + \mu K_s^e)\, D_i/Q_i + (K_r + \mu K_r^e)\, D_i/Q_i + (H_i + \mu H_i^e)\, Q_i/2 \quad (14)$$

(2) if $i \succ k$, then the annual total carbon emissions and costs of the retailer i and supplier are as follows:

$$E_{si}(Q_i) = K_s^e\, D_i/Q_i + K_r^e\, D_i/Q_i + H_s^e\, Q_i/2 \quad (15)$$

$$C_{si}(Q_i) = 0.05(F + BQ_i) + (K_s + \mu K_s^e)\, D_i/Q_i + (K_r + \mu K_r^e)\, D_i/Q_i + (H_s + \mu H_s^e)\, Q_i/2 \quad (16)$$

Proposition 2. In the VC model, the optimal solutions are as follows:

if $i \le k$, then

$$Q_i^{VC*} = \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)D_i/(0.1B + H_i + \mu H_i^e)} \quad (17)$$

if $i \succ k$, then

$$Q_i^{VC*} = \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)D_i/(0.1B + H_s + \mu H_s^e)} \quad (18)$$

For proof, see **Appendix B**.

Substituting the optimal solutions into Equations (13-16), we get all outcomes of this mode. We mainly focus on the performance of overall supply chain in this paper. Therefore, we only present the annual carbon emissions and cost of overall supply chain given the optimal solutions, as follows:

$$\begin{aligned} E_{sc}{}^{VC*} &= \sum_{i=1}^{k} \Big[(K_s^e + K_r^e)\sqrt{(0.1B + H_i + \mu H_i^e)/2(K_s + \mu K_s^e + K_r + \mu K_r^e)} \\ &\quad + H_i^e \sqrt{(K_s + \mu K_s^e + K_r + \mu K_r^e)/2(0.1B + H_i + \mu H_i^e)}\Big]\sqrt{D_i} \\ &\quad + \sum_{i=k+1}^{n} \Big[(K_s^e + K_r^e)\sqrt{(0.1B + H_s + \mu H_s^e)/2(K_s + \mu K_s^e + K_r + \mu K_r^e)} \\ &\quad + H_s^e \sqrt{(K_s + \mu K_s^e + K_r + \mu K_r^e)/2(0.1B + H_s + \mu H_s^e)}\Big]\sqrt{D_i} \end{aligned} \quad (19)$$

$$\begin{aligned} C_{sc}{}^{VC*} &= 0.05(k+1)F \\ &\quad + \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)}\left[\sum_{i=1}^{k}\sqrt{(0.1B + H_i + \mu H_i^e)D_i} + \sum_{i=k+1}^{n}\sqrt{(0.1B + H_s + \mu H_s^e)D_i}\right] \end{aligned} \quad (20)$$

From the optimal solutions of Propositions 1 and 2, we have the following Corollary.

Corollary 1. $Q_i^{VC*} \succ Q_i^{DD*}$, $E_{sc}{}^{VC*} \prec E_{sc}{}^{DD*}$, $C_{sc}{}^{VC*} \prec C_{sc}{}^{DD*}$.

For proof, see **Appendix C**.

Corollary 1 shows that the optimal ordering quantity of retailer i in the VC model is larger than that in the DD model. However, both of the annual carbon emissions and cost of supply chain in the VC model are smaller than that in the DD model. The Corollary suggests that the vertical cooperation among supply chain members is not only beneficial to decrease cost, but also conducive to reduce carbon emissions.

Horizontal Cooperation Model

The retailers order products and manage inventories jointly, and have no cooperation with supplier in HC mode. The multiple retailers build a warehouse in which the annual inventory holding cost per product unit is the minimal. We assume that the supplier's ordering quantity is W_t times that of the retailers, in order to meet demand fluctuation of retailers. The annual total carbon emissions and costs of the retailers and supplier are as follows:

$$E_{rt}(Q_t) = K_r^e \sum_{i \in N} D_i/Q_t + \mu H_t^e\, Q_t/2 \quad (21)$$

$$E_s(W_t, Q_t) = K_s^e \sum_{i \in N} D_i/W_t Q_t + H_s^e\, W_t Q_t/2 \quad (22)$$

$$C_{rt}(Q_t) = 0.05(F + BQ_t) + (K_r + \mu K_r^e) \sum_{i \in N} D_i/Q_t + (H_t + \mu H_t^e)\, Q_t/2 \quad (23)$$

$$C_s(W_t, Q_t) = 0.05(F + BW_t Q_t) + (K_s + \mu K_s^e) \sum_{i \in N} D_i/W_t Q_t + (H_s + \mu H_s^e)\, W_t Q_t/2 \quad (24)$$

where $H_t = \min_{i \in N}\{H_i\}$, $H_t{}^e = \min_{i \in N}\{H_i{}^e\}$.

Proposition 3. In the HC model, the optimal solutions are as follows:

$$Q_t^{HC*} = \sqrt{2(K_r + \mu K_r^e) \sum_{i \in N} D_i/(0.1B + H_t + \mu H_t^e)} \quad (25)$$

$$W_t{}^{HC*} = \frac{1}{Q_t{}^{HC*}} \sqrt{2(K_s + \mu K_s^e) \sum_{i \in N} D_i/(0.1B + H_s + \mu H_s^e)} \quad (26)$$

For proof, see **Appendix D**.

Substituting the optimal solutions into Equations (21-24), we derive all outcomes of this mode. We also only present the annual carbon emissions and cost of overall supply chain given the above optimal solutions, as follows:

$$
\begin{aligned}
&{E_{sc}}^{HC*} \\
&= [K_r^e\sqrt{(0.1B + H_t + \mu H_t^e)/2(K_r + \mu K_r^e)} \\
&+ H_t^e\sqrt{(K_r + \mu K_r^e)/2(0.1B + H_t + \mu H_t^e)} \\
&+ K_s^e\sqrt{(0.1B + H_s + \mu H_s^e)/2(K_s + \mu K_s^e)} \\
&+ H_s^e\sqrt{(K_s + \mu K_s^e)/2(0.1B + H_s + \mu H_s^e)}]\sqrt{\sum_{i\in N} D_i}
\end{aligned} \tag{27}
$$

$$
\begin{aligned}
&{C_{sc}}^{HC*} \\
&= 0.1F + [\sqrt{2(K_r + \mu K_r^e)(0.1B + H_t + \mu H_t^e)} \\
&+ \sqrt{2(K_s + \mu K_s^e)(0.1B + H_s + \mu H_s^e)}]\sqrt{\sum_{i\in N} D_i}
\end{aligned} \tag{28}
$$

From the optimal solutions of Propositions 1 and 3, we have the following Corollary.

Corollary 2. ${E_{sc}}^{HC*} \prec {E_{sc}}^{DD*}$, ${C_{sc}}^{HC*} \prec {C_{sc}}^{DD*}$.

For proof, see **Appendix E**.

Corollary 2 suggests that the horizontal cooperation strategy is also better than the decentralized decision mode in terms of both of cost and carbon emissions.

Vertical and Horizontal Cooperation Model

The supplier and retailers manage inventories jointly with the VHC mode. In horizontal direction, retailers order products and manage inventories jointly. In vertical direction, the supplier and retailers manage inventories jointly and share market demand information. In the supply chain system, there is only one warehouse in which the annual inventory holding cost is the minimal. The supplier does not need to place more orders to meet retailers' demand fluctuation similar to VC mode. Hence, the annual total carbon emissions and cost of the supply chain are as follows:

$$
E_{sc}(Q_t) = K_s^e\sum_{i\in N} D_i/Q_t + K_r^e\sum_{i\in N} D_i/Q_t + H_{vh}^e\, Q_t/2 \tag{29}
$$

$$
\begin{aligned}
C_{sc}(Q_t) &= 0.05(F + BQ_t) + (K_s + \mu K_s^e)\sum_{i\in N} D_i/Q_t \\
&+ (K_r + \mu K_r^e)\sum_{i\in N} D_i/Q_t + (H_{vh} + \mu H_{vh}^e)\, Q_t/2
\end{aligned} \tag{30}
$$

where $H_{vh} = \min_{i\in(N,s)}\{H_i\}$, $H_{vh}^e = \min_{i\in(N,s)}\{{H_i}^e\}$.

Proposition 4. In the VHC model, the optimal solution is as follows:

$$
\begin{aligned}
&Q_t^{VHC*} \\
&= \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)\sum_{i\in N} D_i/(0.1B + H_{vh} + \mu H_{vh}^e)}
\end{aligned} \tag{31}
$$

For proof, see **Appendix F**.

Substituting the optimal solution into Equations (29) and (30), we obtain the annual carbon emissions and cost of overall supply chain given the above optimal solution, as follows:

$$
\begin{aligned}
&{E_{sc}}^{VHC*} \\
&= [(K_s^e + K_r^e)\sqrt{(0.1B + H_{vh} + \mu H_{vh}^e)/2(K_s + \mu K_s^e + K_r + \mu K_r^e)} \\
&+ H_{vh}^e\sqrt{(K_s + \mu K_s^e + K_r + \mu K_r^e)/2(0.1B + H_{vh} + \mu H_{vh}^e)}]\sqrt{\sum_{i\in N} D_i}
\end{aligned} \tag{32}
$$

$$
\begin{aligned}
&{C_{sc}}^{VHC*} \\
&= 0.05F \\
&+ \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)(0.1B + H_{vh} + \mu H_{vh}^e)\sum_{i\in N} D_i}
\end{aligned} \tag{33}
$$

From the optimal solutions of Propositions 2-4, we have the following Corollary.

Corollary 3. (1) $Q_t^{VHC*} \succ Q_t^{HC*}$; (2) ${E_{sc}}^{VHC*} \prec {E_{sc}}^{VC*}$, ${C_{sc}}^{VHC*} \prec {C_{sc}}^{VC*}$; (3) ${E_{sc}}^{VHC*} \prec {E_{sc}}^{HC*}$, ${C_{sc}}^{VHC*} \prec {C_{sc}}^{HC*}$.

For proof, see **Appendix G**.

Corollary 3(1) shows that the optimal joint ordering quantity of retailers in the VHC model is larger than that in the HC model. However, compare with the VC mode, both of the annual carbon emissions and cost of overall supply chain in VHC mode are lower according to Corollary 3(2). The performance of VHC mode is also better than that of HC mode in terms of both of the carbon emissions and cost according to Corollary 3(3).

From the Corollaries 1-3, we can conclude that the vertical and horizontal cooperation mode is always a dominant strategy, with which both of the cost and carbon emissions in supply chain are the minimal.

COLLABORATIVE CONDITION AND COST ALLOCATION IN VHC MODE

The cooperation can be achieved by meeting two conditions: collective and individual rationality, that is, require not only higher benefits in general, but also incentive compatibility among individuals. The Corollaries 1-3 have proved that the VHC mode meets the collective rationality. If the VHC between the supply chain members can achieve Pareto improvement, then, the individual rationality would be fulfilled.

The supplier s and overall retailers N constitute the alliance of supply chain collaboration, which is denoted by $N + s$. We can define the corresponding VHC cost game $(N + s, C)$ as follows. For all coalitions $G \subset N + s$, the cost if $s \in G$, then

$$C(G) = 0.05F + \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)(0.1B + H(G) + \mu H(G)^e) \sum_{i \in G \backslash s} D_i} \quad (34)$$

if $s \notin G$, then

$$C(G) = 0.05F + \sqrt{2(K_r + \mu K_r^e)(0.1B + H(G) + \mu H(G)^e) \sum_{i \in G} D_i} \quad (35)$$

where $H(G) = \min_{i \in G}\{H_i\}$, $H(G)^e = \min_{i \in G}\{H_i{}^e\}$.

Proposition 5. If $0.05F \geq \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)(0.1B + H_s + \mu H_s^e) \sum_{i \in N} D_i}$, then the characteristic function C has strict subadditivity in the VHC cost game $(N + s, C)$.

For proof, see **Appendix H**.

Proposition 5 indicates that if $0.05F \geq \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)(0.1B + H_s + \mu H_s^e) \sum_{i \in N} D_i}$, then the cost of utilizing one warehouse is lower than that of utilizing multiple warehouses for all coalitions $G \subset N + s$. The left side of the above inequality denotes annual fixed construction cost of warehouse, and the right side includes $\sum_{i \in N} D_i$, $K_s + \mu K_s^e$, $K_r + \mu K_r^e$ and $0.1B + H_s + \mu H_s^e$. Therefore, the higher the fixed construction cost of warehouse is, the stronger the enterprises' cooperation motivation is. However, the higher variable construction cost per product unit of warehouse, ordering cost per order of supplier/retailer, carbon emission per order linked to supplier's/retailers' order, annual inventory holding cost per product unit of supplier, annual carbon emission per product unit linked to supplier's holding inventory, carbon tax rate and the larger annual total demand of retailers will hinder cooperation. Proposition 5 implies that the higher carbon tax rate will increase carbon emissions for overall supply chain, which violates the original intention of the government employing carbon tax mechanism.

The cooperative participants are not only concerned with the surplus brought by cooperation, but also more concerned about the distribution of the surplus. Thus, for the successful implementation of VHC mode, we need to find a cost allocation scheme $P = \{P_i, i \in N + s\}$, where P_i denotes the payment of enterprise i, which satisfies collective and individual rationality, that is, $\sum_{i \in N+s} P_i \leq C(N + s)$ and $\sum_{i \in G} P_i \leq C(G)$ for all $G \subset N + s$. This means vector P belongs to the core of the VHC cost game $(N + s, C)$.

The proportion rule is an effective method to solve the problem of ordering and holding cost allocation, which was proposed by Meca et al. (2004). We define an improved proportional rule to allocate the cost of the grand coalition considering supplier's participation based on the proportional rule suggested by Meca et al. (2004). The rule $P(C)$ allocates annual fixed construction cost of warehouse to the supplier and divides the remaining cost of the grand coalition proportionally to the demands for retailers. This means that for s,

$$P_s(C) = 0.05F \quad (36)$$

and for each $i \in N$,

$$P_i(C) = \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)(0.1B + H_{vh} + \mu H_{vh}^e) \sum_{j \in N} D_j}\, D_i \Big/ \sum_{j \in N} D_j \quad (37)$$

Proposition 6. If $0.05F \geq \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)(0.1B + H_s + \mu H_s^e) \sum_{i \in N} D_i}$, then the allocation scheme $P(C)$ belongs to the core of the VHC cost game $(N + s, C)$.

For proof, see **Appendix I**.

If $0.05F \geq \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)(0.1B + H_s + \mu H_s^e) \sum_{i \in N} D_i}$, then the VHC cost game $(N + s, C)$ is subadditive, but not necessarily concave, as the following example shows.

Example 1. Consider the VHC cost situation with players set $N + s = \{s, 1,2,3\}$, $F = 200$, $B = 10$, $K_s + \mu K_s^e = K_r + \mu K_r^e = 0.5$, $H_s + \mu H_s^e = 9$, $H_1 + \mu H_1^e = 9$, $H_2 + \mu H_2^e = 18$, $H_3 + \mu H_3^e = 27$ and $D_1 = D_2 = D_3 = 1$. With these values of parameters, we can easily derive $0.05F \geq \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)\ (0.1B + H_s + \mu H_s^e) \sum_{i \in N} D_i}$, that is, $10 \succ 7.75$. Then,

$C(\{s, 1,3\}) - C(\{1,3\}) = \sqrt{40} - 2\sqrt{20} - 10 \prec 0$ and $C(\{s, 1,2,3\}) - C(\{s, 1,3\}) = \sqrt{60} - \sqrt{40} \succ 0$. So, this cost game is not concave.

Since the VHC cost game is not necessarily concave, there may be marginal vectors that lie outside the core. However we will show that the VHC game is permutationally concave game. Permutationally concave game is introduced by Granot and Huberman (1982) and studied by Driessen (1988). Let $\Pi(N)$ denote the set of all permutations of the player set N. For all $\sigma \in \Pi(N)$, $\sigma(i)$ denotes the position of player $i \in N$. Let $L_i^\sigma = \{\sigma(j): j \leq i\}$ be the set of players whose sort number are not larger than i. Define for all $\sigma \in \Pi(N)$, $\sigma(0) = 0$ and $L_0^\sigma = \emptyset$. If a cost game satisfies $C(L_i{}^\sigma \cup R) - C(L_i{}^\sigma) \geq C(L_j{}^\sigma \cup R) - C(L_j{}^\sigma)$ for all $i, j \in N \cup$

Table 2. Other values of parameters

F	B	K_r	K_r^e	K_s	K_s^e	H_s	H_1	H_2	H_3	H_s^e	H_1^e	H_2^e	H_3^e
20000	10	100	10	80	8	4	4	2	6	2	2	1	3

Table 3. The optimal costs and carbon emissions in four modes

Mode	Cost	Carbon emissions
DD mode	6325.67	436.32
VC mode	3526.94	280.43
HC mode	3176.51	211.93
VHC mode	1722.99	123.24

$\{0\}$, $\sigma(i) \le \sigma(j)$ and $R \subset N \backslash L_j^{\sigma}$, then it can be called permutationally concave with respect to the ordering $\sigma \in \Pi(N)$. If there exists an ordering $\sigma \in \Pi(N)$ and the cost game is permutationally concave with respect to the ordering σ, then the game is called permutationally concave game.

Proposition 7. If $0.05F \ge \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)(0.1B + H_s + \mu H_s^e)\sum_{i\in N} D_i}$, then the VHC cost game is permutationally concave game.

For proof, see **Appendix J**.

Proposition 7 shows there is at least one marginal vector in the core of the VHC cost game $(N + s, C)$.

AN EXAMPLE

In this example, we consider three supermarkets, supermarket1, supermarket2, supermarket3, (in short: 1, 2 and 3), which operate in the same country. The supermarkets order a same commodity from the supplier s. Supermarket1 needs 110 units per year, supermarket2 100 units and supermarket3 120 units per year. The carbon tax rate is 1 dollar per unit. Other values of parameters are shown in **Table 2**.

Substituting the values of parameters into Equations (11), (12), (19), (20), (27), (28), (32) and (33). We can derive the optimal costs and carbon emissions of supply chain in four modes, which are present in **Table 3**.

Table 3 shows that the three cooperation modes all can reduce cost and carbon emissions, and the performance of supply chain is best with VHC mode. Hence, the VHC mode is a dominant strategy, which make the cost and carbon emissions to reduce by 72.76% and 71.75% respectively.

In case the supplier and supermarkets work together, we find that the inventories are held in supermarket2's warehouse, since its annual inventory holding cost per product unit is the lowest. The cost of the various coalitions in the VHC cost game equals (in dollars)

$C(\{s\}) = 2103.63$, $C(\{1\}) = 1411.58$, $C(\{2\}) = 1296.65$, $C(\{3\}) = 1513.81$,

$C(\{s,1\}) = 1552.20$, $C(\{s,2\}) = 1398.00$, $C(\{s,3\}) = 1576.75$, $C(\{1,2\}) = 1429.88$,

$C(\{1,3\}) = 1595.15$, $C(\{2,3\}) = 1440.00$, $C(\{s,1,2\}) = 1576.75$, $C(\{s,1,3\}) = 1798.47$,

$C(\{s,2,3\}) = 1590.32$, $C(\{1,2,3\}) = 1538.89$, $C(\{s,1,2,3\}) = 1722.99$.

Since $0.05F = 1000 > \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)(0.1B + H_s + \mu H_s^e)\sum_{i\in N} D_i} = 957$, we can get the allocation scheme $P(C)$ belongs to the core of the VHC cost game $(N + s, C)$ from Proposition 6. Therefore the rule $P(C)$ assigns the cost \$(1000, 241, 219, 263) to the supplier and supermarkets and the supplier and supermarkets are all reluctant to break away from the supply chain alliance, which will be useful to maintain the giant alliance's stability and reduce the carbon emissions in supply chain.

CONCLUSIONS

In this paper, we consider a supply chain consisting of one supplier and multiple retailers under the carbon tax scheme. The supplier and retailers intend to reduce cost and carbon emissions by utilizing cooperation. We investigate four different decision models: decentralized decision, vertical cooperation, horizontal cooperation and vertical and horizontal cooperation. The work present three main results, which can provide useful insights for supply chain members.

Firstly, after analyzing and comparing the optimal solutions of different models, we find that the VC mode, HC mode and VHC mode are all beneficial to reduce cost and carbon emissions, and VHC mode is always a dominant strategy for the supply chain. Compared with VC mode and HC mode, VHC mode brings lower cost and carbon emissions. Hence, the vertical and horizontal cooperation among supply chain members is beneficial for the supply chain and environment.

Secondly, we investigate the problems of collaborative condition and cost allocation in VHC mode and derive the conclusion that if the fixed construction cost of warehouse is high enough and satisfy the condition $0.05F \geq \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)(0.1B + H_s + \mu H_s^e)\sum_{i\in N} D_i}$, then the characteristic function C has strict subadditivity in the VHC cost game $(N + s, C)$. Although the VHC cost game is not necessarily concave, it is permutationally concave game. Thus, there is at least one marginal vector in the core of the VHC cost game. We design a cost allocation scheme $P(C)$ based on proportion rule, which is proved to belong to the core of the VHC cost game.

Finally, we get an interesting conclusion that higher carbon tax rate will increase carbon emissions for overall supply chain by hindering the vertical and horizontal cooperation among supply chain members, which violates the original intention of the government employing carbon tax mechanism. Therefore, the government should take reasonable carbon tax rate for controlling carbon emissions.

There are several directions for future research. First, we have examined the effects of cooperation among supply chain members on carbon emission reduction under the carbon tax scheme. The cap-and-trade policy is also an effective way to reduce carbon emissions. Thus, one possible extension is to investigate the effects of cooperation on carbon emission reduction under the cap-and-trade scheme. Second, we assume that the retailers' demand is determined in this work. Actually, with the awakening of consumers' low-carbon awareness, the low carbon operation in supply chain will bring greater demand. Hence, we can study the impact of uncertain demand on the decisions in different cooperation models in further.

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APPENDIX A

Proof of Proposition 1. The retailers simultaneously and uncooperatively determine the ordering quantity, and then the supplier determines the multiply given ordering quantity of retailer i. Firstly, we solve for the retailer i's cost function. The first order derivative of $C_i(Q_i)$ with respect to Q_i from Equation (3) can be shown as: $\frac{\partial C_i(Q_i)}{\partial Q_i} = 0.05B - (K_r + \mu K_r^e)\, D_i/{Q_i}^2 + (H_i + \mu H_i^e)/2$. The second order condition is $\frac{\partial^2 C_i(Q_i)}{\partial {Q_i}^2} = 2(K_r + \mu K_r^e)\, D_i/{Q_i}^3 \succ 0$. Thus the retailer i's cost function is strictly convex in Q_i. Equating the first order condition to 0, we can obtain $Q_i^{DD*} = \sqrt{2(K_r + \mu K_r^e)D_i/(0.1B + H_i + \mu H_i^e)}$.

Then, we solve the optimal decision of the supplier. We substitute the value of Q_i^{DD*} into Equation (4) and get ${W_i}^{DD*} = \frac{1}{{Q_i}^{DD*}}\sqrt{2(K_s + \mu K_s^e)D_i/(0.1B + H_s + \mu H_s^e)}$.

APPENDIX B

Proof of Proposition 2. The retailer i and supplier jointly determine the ordering quantity. When $i \leq k$, then the first order derivative of $C_{si}(Q_i)$ with respect to Q_i from Equation (14) can be shown as: $\frac{\partial C_{si}(Q_i)}{\partial Q_i} = 0.05B - (K_s + \mu K_s^e)\, D_i/{Q_i}^2 - (K_r + \mu K_r^e)\, D_i/{Q_i}^2 + (H_i + \mu H_i^e)/2$. The second order condition can be shown as: $\frac{\partial^2 C_{si}(Q_i)}{\partial {Q_i}^2} = 2(K_s + \mu K_s^e)\, D_i/{Q_i}^3 + 2(K_r + \mu K_r^e)\, D_i/{Q_i}^3 \succ 0$. Equating the first order condition to 0, we can obtain $Q_i^{VC*} = \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)D_i/(0.1B + H_i + \mu H_i^e)}$.

When $i \succ k$, the proof is similar to that of the case which $i \leq k$, so we omit it.

APPENDIX C

Proof of Corollary 1. If $i \succ k$, then $H_s \leq H_i$. Hence, we can get $Q_i^{VC*} \succ Q_i^{DD*}$ from the expressions of optimal solutions directly.

For all $i, j \in (N, s)$, if $H_i \leq H_j$, then $H_i^e \leq H_j^e$. So we can get $H_1^e \leq H_2^e \leq \cdots \ldots \leq H_k^e \prec H_s^e \leq H_{k+1}^e \leq \cdots \ldots \leq H_n^e$, when $H_1 \leq H_2 \leq \cdots \ldots \leq H_k \prec H_s \leq H_{k+1} \leq \cdots \ldots \leq H_n$. Because of $\sqrt{K_s + \mu K_s^e + K_r + \mu K_r^e} \prec \sqrt{K_s + \mu K_s^e} + \sqrt{K_r + \mu K_r^e}$, we can obtain

$$\sum_{i=1}^{k} H_i^e\sqrt{(K_s + \mu K_s^e + K_r + \mu K_r^e)D_i/2(0.1B + H_i + \mu H_i^e)} + \sum_{i=k+1}^{n} H_s^e\sqrt{(K_s + \mu K_s^e + K_r + \mu K_r^e)D_i/2(0.1B + H_s + \mu H_s^e)} \prec \sum_{i=1}^{k} H_i^e\sqrt{(K_s + \mu K_s^e)D_i/2(0.1B + H_i + \mu H_i^e)} + \sum_{i=1}^{k} H_i^e\sqrt{(K_r + \mu K_r^e)D_i/2(0.1B + H_i + \mu H_i^e)} + \sum_{i=k+1}^{n} H_s^e\sqrt{(K_s + \mu K_s^e)D_i/2(0.1B + H_s + \mu H_s^e)} + \sum_{i=k+1}^{n} H_s^e\sqrt{(K_r + \mu K_r^e)D_i/2(0.1B + H_s + \mu H_s^e)} \prec \sum_{i\in N} H_s^e\sqrt{(K_s + \mu K_s^e)D_i/2(0.1B + H_s + \mu H_s^e)} + \sum_{i=1}^{k} H_i^e\sqrt{(K_r + \mu K_r^e)D_i/2(0.1B + H_i + \mu H_i^e)} + \sum_{i=k+1}^{n} H_s^e\sqrt{(K_r + \mu K_r^e)D_i/2(0.1B + H_s + \mu H_s^e)} \leq \sum_{i\in N} H_i^e\sqrt{(K_r + \mu K_r^e)D_i/2(0.1B + H_i + \mu H_i^e)} + H_s^e\sqrt{(K_s + \mu K_s^e)/2(0.1B + H_s + \mu H_s^e)}\sum_{i\in N}\sqrt{D_i}.$$

Thus, from Equations (11) and (19), we can derive

$${E_{sc}}^{VC*} = \sum_{i=1}^{k} K_r^e\sqrt{(0.1B + H_i + \mu H_i^e)D_i/2(K_s + \mu K_s^e + K_r + \mu K_r^e)} + \sum_{i=k+1}^{n} K_r^e\sqrt{(0.1B + H_s + \mu H_s^e)D_i/2(K_s + \mu K_s^e + K_r + \mu K_r^e)} + \sum_{i=1}^{k} K_s^e\sqrt{(0.1B + H_i + \mu H_i^e)D_i/2(K_s + \mu K_s^e + K_r + \mu K_r^e)} + \sum_{i=k+1}^{n} K_s^e\sqrt{(0.1B + H_s + \mu H_s^e)D_i/2(K_s + \mu K_s^e + K_r + \mu K_r^e)} + \sum_{i=1}^{k} H_i^e\sqrt{(K_s + \mu K_s^e + K_r + \mu K_r^e)D_i/2(0.1B + H_i + \mu H_i^e)} + \sum_{i=k+1}^{n} H_s^e\sqrt{(K_s + \mu K_s^e + K_r + \mu K_r^e)D_i/2(0.1B + H_s + \mu H_s^e)} \prec \sum_{i=1}^{k} K_r^e\sqrt{(0.1B + H_i + \mu H_i^e)D_i/2(K_s + \mu K_s^e + K_r + \mu K_r^e)} + \sum_{i=k+1}^{n} K_r^e\sqrt{(0.1B + H_s + \mu H_s^e)D_i/2(K_s + \mu K_s^e + K_r + \mu K_r^e)} + K_s^e\sqrt{(0.1B + H_s + \mu H_s^e)/2(K_s + \mu K_s^e + K_r + \mu K_r^e)}\sum_{i\in N}\sqrt{D_i} + \sum_{i=1}^{k} H_i^e\sqrt{(K_s + \mu K_s^e + K_r + \mu K_r^e)D_i/2(0.1B + H_i + \mu H_i^e)} +$$

$\sum_{i=k+1}^{n} H_s^e \sqrt{(K_s + \mu K_s^e + K_r + \mu K_r^e)D_i / 2(0.1B + H_s + \mu H_s^e)} < \sum_{i=1}^{k}[K_r^e \sqrt{(0.1B + H_i + \mu H_i^e)/2(K_r + \mu K_r^e)} + H_i^e \sqrt{(K_r + \mu K_r^e)/2(0.1B + H_i + \mu H_i^e)}] \sqrt{D_i} + \sum_{i=k+1}^{n}[K_r^e \sqrt{(0.1B + H_i + \mu H_i^e)/2(K_r + \mu K_r^e)} + H_i^e \sqrt{(K_r + \mu K_r^e)/2(0.1B + H_i + \mu H_i^e)}] \sqrt{D_i} + [K_s^e \sqrt{(0.1B + H_s + \mu H_s^e)/2(K_s + \mu K_s^e)} + H_s^e \sqrt{(K_s + \mu K_s^e)/2(0.1B + H_s + \mu H_s^e)}] \sum_{i\in N} \sqrt{D_i} = E_{sc}{}^{DD*}$;

$$C_{sc}{}^{VC*} = 0.05(k+1)F + \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)}[\sum_{i=1}^{k} \sqrt{(0.1B + H_i + \mu H_i^e)D_i} + \sum_{i=k+1}^{n} \sqrt{(0.1B + H_s + \mu H_s^e)D_i}]$$

From Equations (12) and (20), we can derive

$C_{sc}{}^{VC*} = 0.05(k+1)F + \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)}[\sum_{i=1}^{k} \sqrt{(0.1B + H_i + \mu H_i^e)D_i} + \sum_{i=k+1}^{n} \sqrt{(0.1B + H_s + \mu H_s^e)D_i}] < 0.05(k+1)F + \sqrt{2(K_s + \mu K_s^e)}[\sum_{i=1}^{k} \sqrt{(0.1B + H_i + \mu H_i^e)D_i} + \sum_{i=k+1}^{n} \sqrt{(0.1B + H_s + \mu H_s^e)D_i}] + \sqrt{2(K_r + \mu K_r^e)}[\sum_{i=1}^{k} \sqrt{(0.1B + H_i + \mu H_i^e)D_i} + \sum_{i=k+1}^{n} \sqrt{(0.1B + H_s + \mu H_s^e)D_i}] < 0.05(n+1)F + \sum_{i\in N} \sqrt{2(K_r + \mu K_r^e)(0.1B + H_i + \mu H_i^e)D_i} + \sqrt{2(K_s + \mu K_s^e)(0.1B + H_s + \mu H_s^e)} \sum_{i\in N} \sqrt{D_i} = C_{sc}{}^{DD*}$.

APPENDIX D

Proof of Proposition 3. The retailers cooperatively determine the joint ordering quantity, and then the supplier determines the multiply given joint ordering quantity. The proof is similar to that of Proposition 1, so we omit it.

APPENDIX E

Proof of Corollary 2. Note that $H_t \le H_i$ and $H_t{}^e \le H_i{}^e$ due to $H_t = \min_{i\in N}\{H_i\}$ and $H_t{}^e = \min_{i\in N}\{H_i{}^e\}$. It is also easily proved that $\sum_{i\in N} \sqrt{D_i} > \sqrt{\sum_{i\in N} D_i}$. Therefore, we can get

$C_{sc}{}^{DD*} = 0.05(n+1)F + \sum_{i\in N} \sqrt{2(K_r + \mu K_r^e)(0.1B + H_i + \mu H_i^e)D_i} + \sqrt{2(K_s + \mu K_s^e)(0.1B + H_s + \mu H_s^e)} \sum_{i\in N} \sqrt{D_i} > 0.1F + [\sqrt{2(K_r + \mu K_r^e)(0.1B + H_t + \mu H_t^e)} + \sqrt{2(K_s + \mu K_s^e)(0.1B + H_s + \mu H_s^e)}]\sqrt{\sum_{i\in N} D_i} = C_{sc}{}^{HC*}$.

For all $i, j \in (N, s)$, if $H_i \le H_j$, then $H_i^e \le H_j^e$. So we can derive

$E_{sc}{}^{DD*} = \sum_{i\in N}[K_r^e \sqrt{(0.1B + H_i + \mu H_i^e)/2(K_r + \mu K_r^e)} + H_i^e \sqrt{(K_r + \mu K_r^e)/2(0.1B + H_i + \mu H_i^e)}]\sqrt{D_i} + [K_s^e \sqrt{(0.1B + H_s + \mu H_s^e)/2(K_s + \mu K_s^e)} + H_s^e \sqrt{(K_s + \mu K_s^e)/2(0.1B + H_s + \mu H_s^e)}] \sum_{i\in N} \sqrt{D_i} \ge [K_r^e \sqrt{(0.1B + H_t + \mu H_t^e)/2(K_r + \mu K_r^e)} + H_t^e \sqrt{(K_r + \mu K_r^e)/2(0.1B + H_t + \mu H_t^e)} + K_s^e \sqrt{(0.1B + H_s + \mu H_s^e)/2(K_s + \mu K_s^e)} + H_s^e \sqrt{(K_s + \mu K_s^e)/2(0.1B + H_s + \mu H_s^e)}] \sum_{i\in N} \sqrt{D_i} > [K_r^e \sqrt{(0.1B + H_t + \mu H_t^e)/2(K_r + \mu K_r^e)} + H_t^e \sqrt{(K_r + \mu K_r^e)/2(0.1B + H_t + \mu H_t^e)} + K_s^e \sqrt{(0.1B + H_s + \mu H_s^e)/2(K_s + \mu K_s^e)} + H_s^e \sqrt{(K_s + \mu K_s^e)/2(0.1B + H_s + \mu H_s^e)}]\sqrt{\sum_{i\in N} D_i} = E_{sc}{}^{HC*}$.

APPENDIX F

Proof of Proposition 4. The supplier and retailers cooperatively determine the joint ordering quantity. The proof is similar to that of Proposition 2, so we omit it.

APPENDIX G

Proof of Corollary 3. (1) Note that $H_t \geq H_{vh}$ and $H_t{}^e \geq H_{vh}{}^e$ due to $H_t = \min_{i\in N}\{H_i\}$, $H_t{}^e = \min_{i\in N}\{H_i{}^e\}$, $H_{vh} = \min_{i\in (N,\ s)}\{H_i\}$ and $H_{vh}{}^e = \min_{i\in(N,s)}\{H_i{}^e\}$. Hence, we can get $Q_t^{VHC*} \succ Q_t^{HC*}$ from the expressions of optimal solutions directly.

(2) Note that $H_i \geq H_{vh}$, $H_i{}^e \geq H_{vh}{}^e$, $H_s \geq H_{vh}$ and $H_s{}^e \geq H_{vh}{}^e$ due to $H_{vh} = \min_{i\in (N,\ s)}\{H_i\}$ and $H_{vh}{}^e = \min_{i\in(N,s)}\{H_i{}^e\}$. For all $i,j \in (N,s)$, if $H_i \leq H_j$, then $H_i^e \leq H_j^e$. Because of $\sum_{i\in N}\sqrt{D_i} \succ \sqrt{\sum_{i\in N} D_i}$, we can get $E_{sc}{}^{VHC*} \prec E_{sc}{}^{VC*}$ and $C_{sc}{}^{VHC*} \prec C_{sc}{}^{VC*}$ from the expressions of optimal solutions directly.

(3) $E_{sc}{}^{HC*} = [K_r^e\sqrt{(0.1B + H_t + \mu H_t^e)/2(K_r + \mu K_r^e)} + H_t^e\sqrt{(K_r + \mu K_r^e)/2(0.1B + H_t + \mu H_t^e)} + K_s^e\sqrt{(0.1B + H_s + \mu H_s^e)/2(K_s + \mu K_s^e)} + H_s^e\sqrt{(K_s + \mu K_s^e)/2(0.1B + H_s + \mu H_s^e)}]\sqrt{\sum_{i\in N} D_i} \geq [K_r^e\sqrt{(0.1B + H_{vh} + \mu H_{vh}^e)/2(K_r + \mu K_r^e)} + H_{hv}^e\sqrt{(K_r + \mu K_r^e)/2(0.1B + H_{vh} + \mu H_{vh}^e)} + K_s^e\sqrt{(0.1B + H_{vh} + \mu H_{vh}^e)/2(K_s + \mu K_s^e)} + H_{vh}^e\sqrt{(K_s + \mu K_s^e)/2(0.1B + H_{vh} + \mu H_{vh}^e)}]\sqrt{\sum_{i\in N} D_i} \succ [(K_s^e + K_r^e)\sqrt{(0.1B + H_{vh} + \mu H_{vh}^e)/2(K_s + \mu K_s^e + K_r + \mu K_r^e)} + H_{vh}^e\sqrt{(K_s + \mu K_s^e + K_r + \mu K_r^e)/2(0.1B + H_{vh} + \mu H_{vh}^e)}]\sqrt{\sum_{i\in N} D_i} = E_{sc}{}^{VHC*} C_{sc}{}^{HC*} = 0.1F + [\sqrt{2(K_r + \mu K_r^e)(0.1B + H_t + \mu H_t^e)} + \sqrt{2(K_s + \mu K_s^e)(0.1B + H_s + \mu H_s^e)}]\sqrt{\sum_{i\in N} D_i} \geq 0.1F + [\sqrt{2(K_r + \mu K_r^e)(0.1B + H_{vh} + \mu H_{vh}^e)} + \sqrt{2(K_s + \mu K_s^e)(0.1B + H_{vh} + \mu H_{vh}^e)}]\sqrt{\sum_{i\in N} D_i} \succ 0.05F + \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)(0.1B + H_{vh} + \mu H_{vh}^e)\sum_{i\in N} D_i} = C_{sc}{}^{VHC*}$.

APPENDIX H

Proof of Proposition 5. In cost game $(N+s, C)$, the cost function $C(G)$ has subadditivity when it satisfies the condition: for all $G, M \subset N+s$, and $G \cap M = \emptyset$, we have that $C(G \cup M) \leq C(G) + C(M)$.

(1) If $G, M \subset N+s$, $G \cap M = \emptyset$, $s \notin G$ and $s \notin M$, then

$C(G \cup M) = 0.05F + \sqrt{2(K_r + \mu K_r^e)(0.1B + H(G\cup M) + \mu H(G\cup M)^e)\sum_{i\in G\cup M} D_i}$,

$C(G) + C(M) = 0.1F + \sqrt{2(K_r + \mu K_r^e)(0.1B + H(G) + \mu H(G)^e)\sum_{i\in G} D_i} + \sqrt{2(K_r + \mu K_r^e)(0.1B + H(M) + \mu H(M)^e)\sum_{i\in M} D_i}$,

Note that $H(G) \geq H(G\cup M)$, $H(M) \geq H(G\cup M)$, $H(G)^e \geq H(G\cup M)^e$ and $H(M)^e \geq H(G\cup M)^e$ due to $G, M \subset N+s$ and $G \cap M = \emptyset$. Obviously, $\sqrt{\sum_{i\in G\cup M} D_i} \prec \sqrt{\sum_{i\in G} D_i} + \sqrt{\sum_{i\in M} D_i}$. So, we can get $C(G\cup M) \prec C(G) + C(M)$.

(2) If $G, M \subset N+s$, $G \cap M = \emptyset$, $s \in G$ and $s \notin M$, then

$C(G\cup M) - [C(G) + C(M)] = -0.05F + \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)(0.1B + H(G\cup M) + \mu H(G\cup M)^e)\sum_{i\in G\cup M\backslash s} D_i} - [\sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)(0.1B + H(G) + \mu H(G)^e)\sum_{i\in G\backslash s} D_i} + \sqrt{2(K_r + \mu K_r^e)(0.1B + H(M) + \mu H(M)^e)\sum_{i\in M} D_i}]$. Note that $H_s \geq H(G\cup M)$ and $H_s{}^e \geq H(G\cup M)^e$ due to $G, M \subset N+s$, $G \cap M = \emptyset$, $s \in G$ and $s \notin M$. So, $\sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)(0.1B + H(G\cup M) + \mu H(G\cup M)^e)\sum_{i\in G\cup M\backslash s} D_i} \leq \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)(0.1B + H_s + \mu H_s^e)\sum_{i\in N} D_i}$.

Therefore, if $0.05F \geq \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)(0.1B + H_s + \mu H_s^e)\sum_{i\in N} D_i}$, then $C(G\cup M) - [C(G) + C(M)] \prec 0$, that is, $C(G\cup M) \prec C(G) + C(M)$.

(3) If $G, M \subset N+s$, $G \cap M = \emptyset$, $s \notin G$ and $s \in M$, then the proof is similar to that of (2), so we omit it.

APPENDIX I

Proof of Proposition 6. By definition of the rule $P(C)$, it holds that $\sum_{i\in N+s} P_i(C) = 0.05F + \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)(0.1B + H_{vh} + \mu H_{vh}^e)\sum_{i\in N} D_i} = C(N+s)$. It also holds that for all $G \subset N + s$,

(1) If $s \in G$, then $H(G) \geq H_{vh}$ and $H(G)^e \geq {H_{vh}}^e$. Hence,

$$\sum_{i\in G} P_i(C) = 0.05F + \sum_{i\in G\backslash s} D_i \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)(0.1B + H_{vh} + \mu H_{vh}^e)/\sum_{j\in N} D_j}$$

$$\prec 0.05F + \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)(0.1B + H(G) + \mu H(G)^e)\sum_{i\in G} D_i} = C(G)$$

(2) If $s \notin G$, then $H_s \geq H_{vh}$ and ${H_s}^e \geq {H_{vh}}^e$. If

$0.05F \geq \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)(0.1B + H_s + \mu H_s^e)\sum_{i\in N} D_i}$, then

$\sum_{i\in G} P_i(C) = \sum_{i\in G} D_i \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)(0.1B + H_{vh} + \mu H_{vh}^e)/\sum_{j\in N} D_j} \prec \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)(0.1B + H_s + \mu H_s^e)\sum_{i\in N} D_i} \prec 0.05F + \sqrt{2(K_r + \mu K_r^e)(0.1B + H(G) + \mu H(G)^e)\sum_{i\in G} D_i} = C(G)$.

Hence, $P(C)$ is a core-allocation of the VHC cost game $(N + s, C)$.

APPENDIX J

Proof of Proposition 7. Since the retailers must order products from the supplier, we number the supplier as 1 and the retailers from 2 to $n+1$, in such a way that the annual inventory holding cost per product unit of all retailers forms a non-decreasing sequence, i.e., ${H_2}^e \leq {H_3}^e \dots\dots \leq {H_{n+1}}^e$. Note that ${H_2}^e \leq {H_3}^e \dots\dots \leq {H_{n+1}}^e$ due to the assumption that for all $i, j \in (N, s)$, if $H_i \leq H_j$, then $H_i^e \leq H_j^e$. Take $\sigma \in \Pi\,(N+s)$ such that $\sigma(i) = i$ for all $i \in N + s$. If $i = 1$, then ${L_1}^\sigma = \sigma(1)$, there are only supplier in the alliance. Since there are not retailers, the supplier does not have orders, and the supplier's cost is mainly the annual fixed construction cost of warehouse, that is, $C({L_1}^\sigma) = 0.05F$. If the VHC cost game satisfies $C({L_i}^\sigma \cup R) - C({L_i}^\sigma) \geq C({L_j}^\sigma \cup R) - C({L_j}^\sigma)$ for all $i, j \in (N + s) \cup \{0\}$, $\sigma(i) \leq \sigma(j)$ and $R \subset (N+s)\backslash {L_j}^\sigma$, then the cost game is permutationally concave game. Let $H_1 = \min\{H_s, H_2\}$ and ${H_1}^e = \min\{{H_s}^e, {H_2}^e\}$. Since $R \subset (N+s)\backslash {L_j}^\sigma$, we can derive $H_1 \leq H(R)$, $H_1 \leq H_s$, ${H_1}^e \leq H(R)^e$ and ${H_1}^e \leq {H_s}^e$. We distinguish five cases.

(1) If $i = j = 0$, then ${L_i}^\sigma = {L_j}^\sigma = \emptyset$ and $C({L_i}^\sigma \cup R) - C({L_i}^\sigma) = C(R) - C(\emptyset) = C({L_j}^\sigma \cup R) - C({L_j}^\sigma)$

(2) If $i = 0$, $j = 1$ and $0.05F \geq \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)(0.1B + H_s + \mu H_s^e)\sum_{i\in N} D_i}$, then ${L_i}^\sigma = \emptyset$, ${L_j}^\sigma = \sigma(1)$, $H_s \geq H({L_j}^\sigma \cup R)$, ${H_s}^e \geq H({L_j}^\sigma \cup R)^e$ and

$$C({L_i}^\sigma \cup R) + C({L_j}^\sigma) = 0.1F + \sqrt{2(K_r + \mu K_r^e)(0.1B + H(R) + \mu H(R)^e)\sum_{i\in R} D_i}$$

$$\succ 0.05F + \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)(0.1B + H({L_j}^\sigma \cup R) + \mu H({L_j}^\sigma \cup R)^e)\sum_{i\in {L_j}^\sigma \cup R\backslash s} D_i}$$

$$= C({L_j}^\sigma \cup R) + C({L_i}^\sigma)$$

(3) If $i = 0$, $j \succ 1$ and $0.05F \geq \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)(0.1B + H_s + \mu H_s^e)\sum_{i\in N} D_i}$, then ${L_i}^\sigma = \emptyset$ and $C({L_i}^\sigma \cup R) + C({L_j}^\sigma) = 0.1F + \sqrt{2(K_r + \mu K_r^e)(0.1B + H(R) + \mu H(R)^e)\sum_{i\in R} D_i} + \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)(0.1B + H_1 + \mu {H_1}^e)\sum_{i\in {L_j}^\sigma} D_i} \succ 0.05F + \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)(0.1B + H_1 + \mu {H_1}^e)\sum_{i\in {L_j}^\sigma \cup R\backslash s} D_i} = C({L_j}^\sigma \cup R) + C({L_i}^\sigma)$

(4) If $i = 1$ and $j \succ 1$, then $H({L_1}^{\sigma} \cup R) \geq H_1$, $H({L_1}^{\sigma} \cup R)^e \geq {H_1}^e$ and

$$C({L_i}^{\sigma} \cup R) + C({L_j}^{\sigma})$$

$$= 0.1F + \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)(0.1B + H({L_1}^{\sigma} \cup R) + \mu H({L_1}^{\sigma} \cup R)^e) \sum_{i \in {L_1}^{\sigma} \cup R \setminus s} D_i}$$

$$+ \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)(0.1B + H_1 + \mu {H_1}^e) \sum_{i \in {L_j}^{\sigma} \setminus s} D_i}$$

$$\geq 0.1F + \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)(0.1B + H_1 + \mu {H_1}^e)}(\sqrt{\sum_{i \in {L_1}^{\sigma} \cup R \setminus s} D_i} + \sqrt{\sum_{i \in {L_j}^{\sigma} \setminus s} D_i})$$

$$\succ 0.1F + \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)(0.1B + H_1 + \mu {H_1}^e) \sum_{i \in {L_j}^{\sigma} \cup R \setminus s} D_i} = C({L_j}^{\sigma} \cup R) + C({L_i}^{\sigma})$$

(5) If $1 \prec i \leq j \leq n + 1$, then

$$C({L_i}^{\sigma} \cup R) + C({L_j}^{\sigma}) = 0.1F + \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)(0.1B + H_1 + \mu {H_1}^e)}(\sqrt{\sum_{i \in {L_i}^{\sigma} \cup R \setminus s} D_i} + \sqrt{\sum_{i \in {L_j}^{\sigma} \setminus s} D_i})$$

$$\succ 0.1F + \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)(0.1B + H_1 + \mu {H_1}^e)}(\sqrt{\sum_{i \in {L_j}^{\sigma} \cup R \setminus s} D_i} + \sqrt{\sum_{i \in {L_i}^{\sigma} \setminus s} D_i})$$

$$= C({L_j}^{\sigma} \cup R) + C({L_i}^{\sigma})$$

So, given $0.05F \geq \sqrt{2(K_s + \mu K_s^e + K_r + \mu K_r^e)(0.1B + H_s + \mu H_s^e) \sum_{i \in N} D_i}$, the VHC cost game is permutationally concave game.